Design and analysis of multi-stable structures

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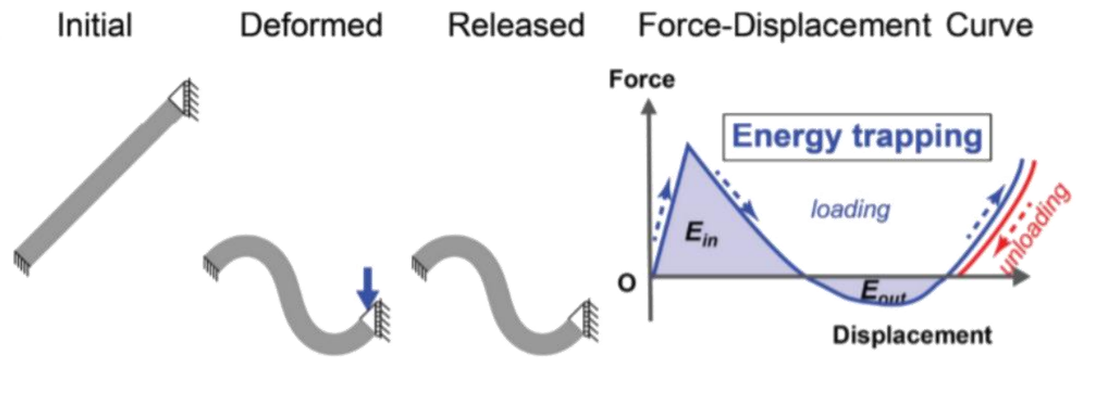
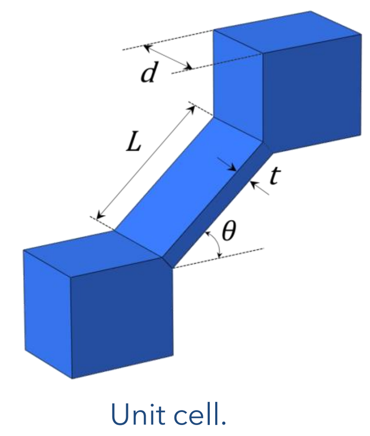
Abstract: In this work we have numerically analyzed the multistability process in metastructures of different topologies: rows, plates and cylinders. For this purpose, 3D numerical models have been developed using Abaqus/CAE and validated with results from the scientific literature, studying the influence of the different geometrical parameters that define these metastructures on their elastic energy absorption capacity. Furthermore, the comparison between quasi-static and dynamic behavior has been carried out, establishing remarkable differences in the stabilization process. Finally, the auxetic behavior of plate-type metastructures has been studied, obtaining the value corresponding to their apparent Poisson's ratio. The results show the influence of the thickness, number of heights and number of unit cells that conform the different topologies on their mechanical response and the amount of energy they can store. In addition, it is shown how dynamic effects modify the sequence in which the different levels reach new stable equilibrium positions.

1. Description of the Problem

The problem analyzed consists of the uniaxial compression of multistable metastructures which allow multiple equilibrium states, both in quasi-static and dynamic regimes. This kind of structures act as an energy-trapping mechanism which depends solely on the (reversible) change in state of prescribed structural geometries. All considered topologies are based on the unit cell show in the following figure, based on a constrained, tilted elastic beam: a bistable elastic element which has been studied by other authors [1,2].



Bistable behaviour. Energy-trapping mechanism [1]



Unit cell. Geometrical parameters

1.1. Polydimethylsiloxane (PDMS):

The material considered in the analysis is PDMS, an elastomeric material valid for its use in additive manufacturing processes, a technology that offers a convenient method for rapidly fabricating materials composed of arrays of tilted elastic beams in programmable motifs.

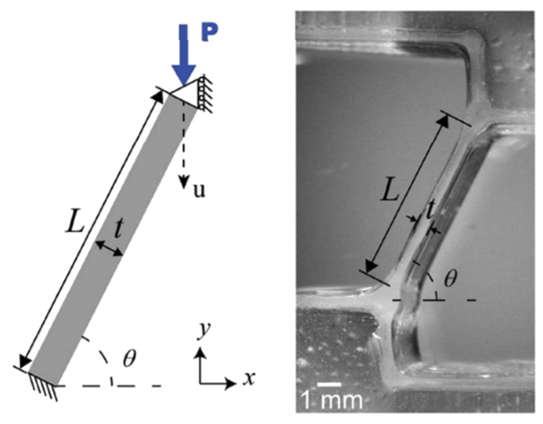
Material Properties →

Density [3]:	$\rho = 982\ kg/m^3$
Elastic Modulus [1]:	$E = 0{,}9568\ MPa$
Poisson coefficient [4]:	$\nu = 0{,}495$

→ PDMS was considered as a linear elastic and quasi-incompressible material.

1.2. Validation of the model:

The validation of the material and mesh properties for the developed numerical model was made in terms of Stored Energy, by comparison with published experimental results [1], considering three geometrical configurations:



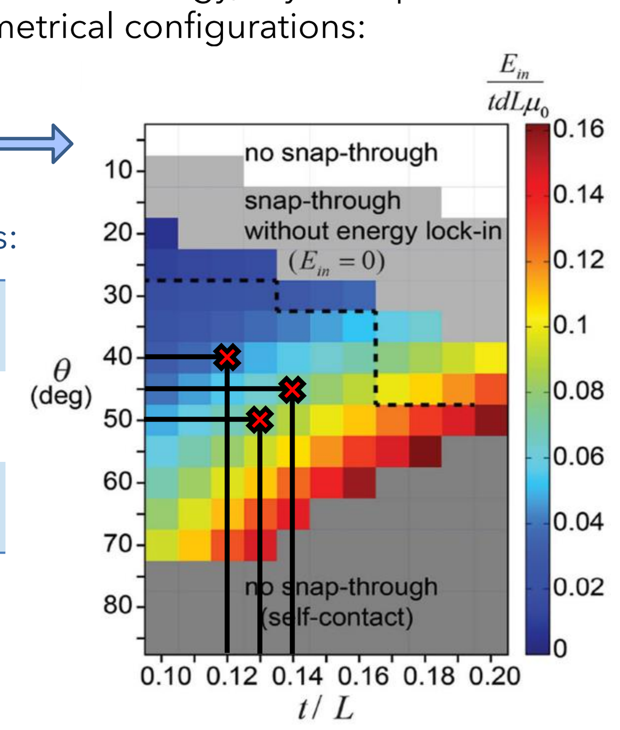
Reference model [1]

Three configurations:

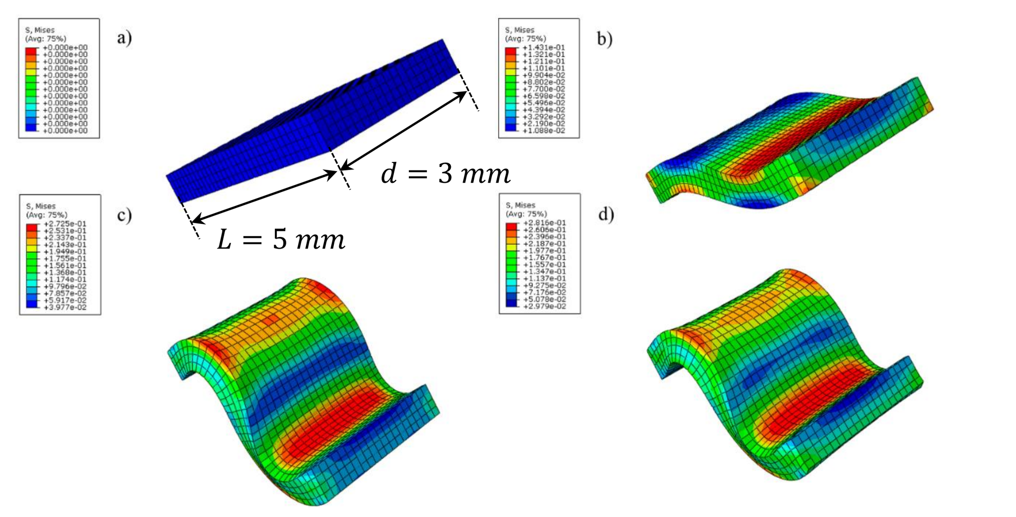
$\frac{t}{L} = 0.12; \theta = 40°$

$\frac{t}{L} = 0.14; \theta = 45°$

$\frac{t}{L} = 0.13; \theta = 50°$



Mechanical response of a constrained, tilted elastic beam [1]



Developed numerical model for validation

Config. 1:	7.6 %
Config. 2:	8.3 %
Config. 3:	9.6 %

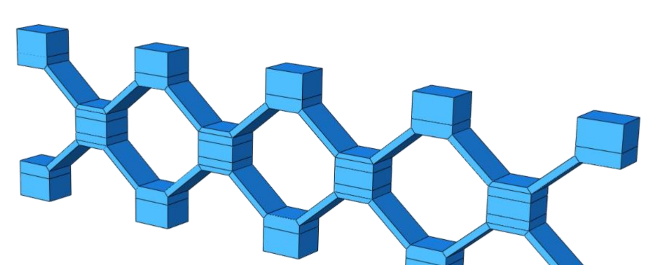
Relative error in terms of Stored Energy

MODEL VALIDATED

2. Topologies under consideration

Three-dimensional numerical models of three different topologies were developed: rows, plates and cylinders. All these geometries were modeled based on the unit cell shown in previous section, fixing the parameters: $L = 5\ mm,\ t = 0.6\ mm,\ \theta = 40°$.

Row-type metastructure with two heights



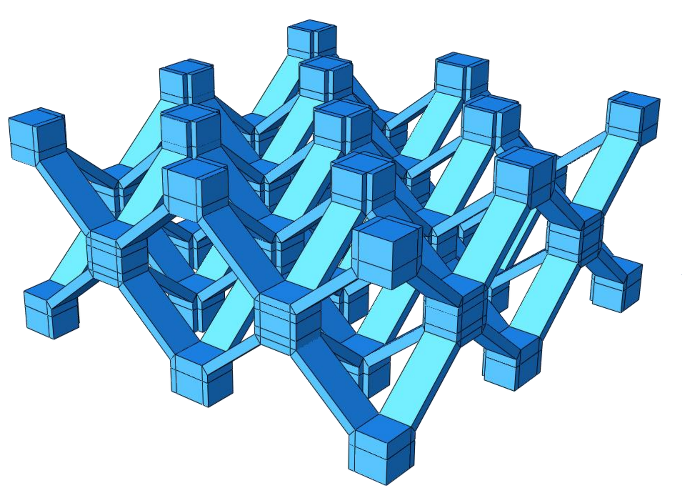
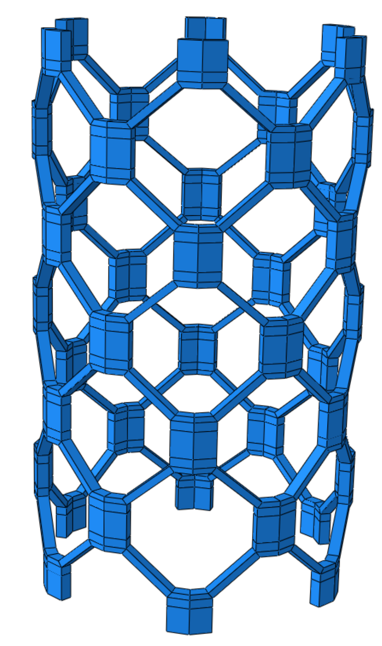


Plate-type metastructure with two heights and four unit cells per side



Cylinder-type metastructure with six heights and twelve sides

2.1. Results:

As an example, the results of applied force and stored internal energy of plate-type metastructures, in which both the number of heights and the thickness of the base unit cell have been varied, are shown.

Thickness: d [mm]	Heights	Internal Energy [mJ]	Force [N]
2	1	2.533	1.682
	2	5.127	1.494
	4	10.032	1.653
3	1	4.219	2.750
	2	8.439	2.735
	4	17.263	2.771
4	1	5.201	3.534
	2	10.449	3.412
	4	22.064	3.672

Internal Energy increases with unit cell thickness and number of heights

Applied force increases with the number of heights but remains constant with thickness

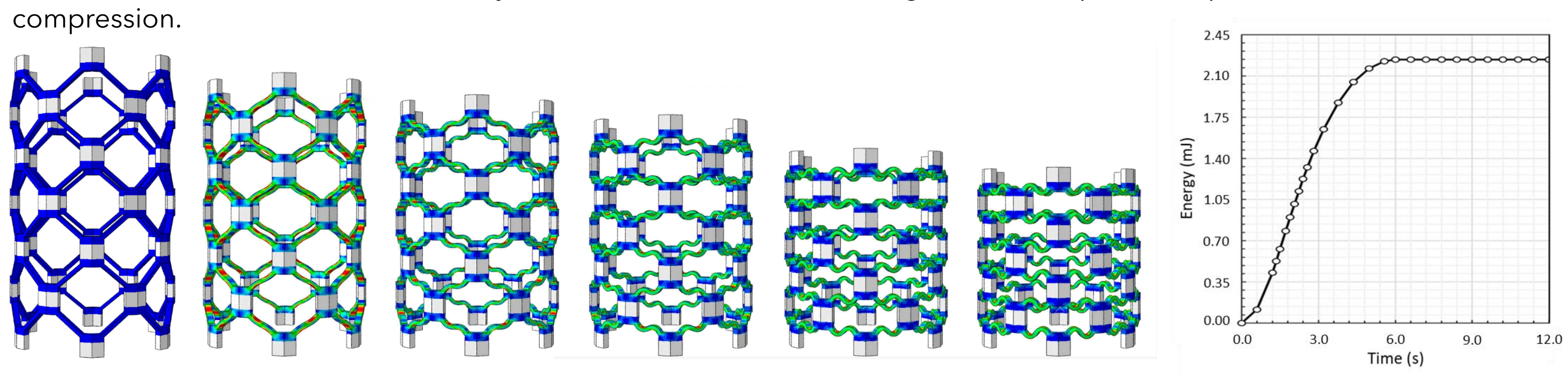
In a generic way, the Internal Energy increases with the increment of the value of each of the geometrical parameters analyzed:

- The correlation is directly proportional to the number of sides of a cylinder-type metastructure, to the number of heights and to the number of unit cells per side.
- The correlation is nonlinearly proportional to unit cell thickness.

3. Quasi-static vs. dynamic behavior

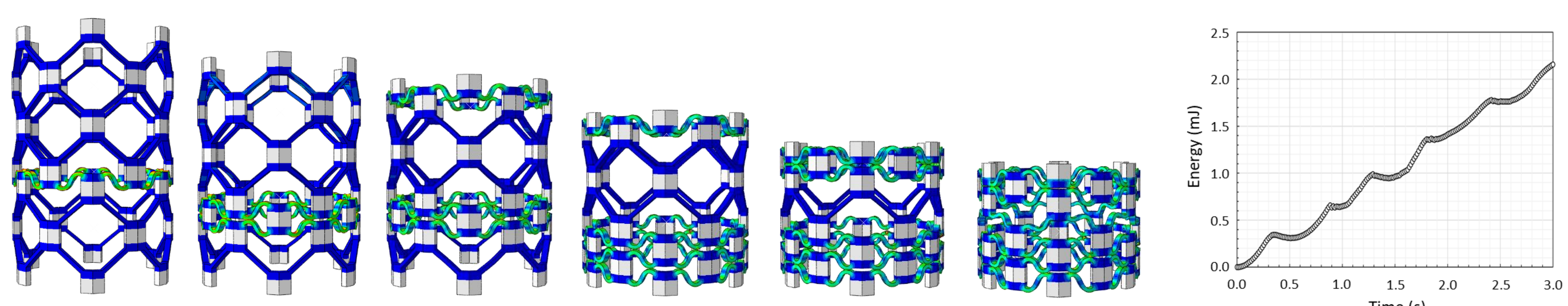
3.1. Quasi-static behavior:

During compression process, it can be seen how the different heights of the metastructure deform in the same way and at the same time, reaching to a new equilibrium position at the end of the compression.



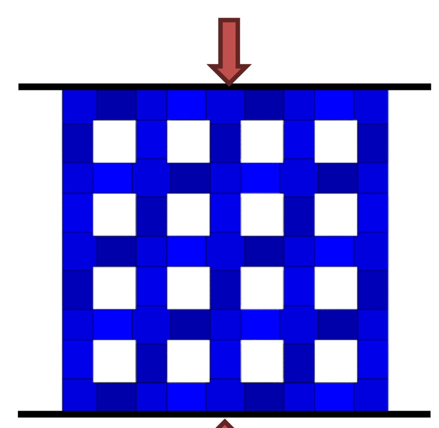
3.2. Dynamic behavior:

In this case, it is possible to observe the different time instants which the collapse of each of the levels of the cylindrical metastructure takes place in. It is also interesting that the successive collapses do not always occur between contiguous heights because of the influence of the wave propagation phenomena.

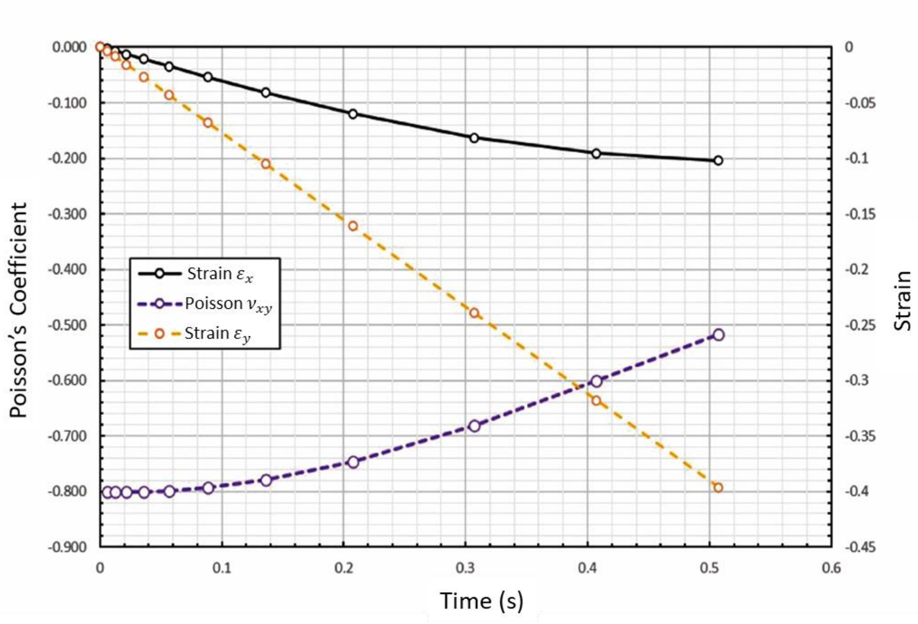


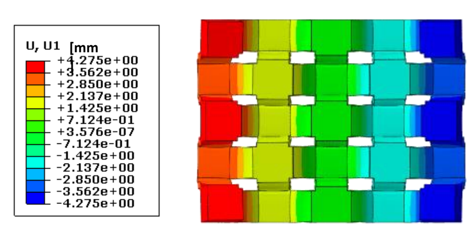
4. Auxetic response for plate-type metastructures

In order to carry out a quantitative analysis of the auxetic behavior, the plate-type metastructure is compressed in the transverse direction to that which reproduces the multistable behavior:



Pre-compression state





Post-compression state

The longitudinal (ε_x) and transverse (ε_y) global strains are represented, as well as the Poisson's coefficient by means of the quotient between the two mentioned strains. Poisson's coefficient is effectively negative (auxetic behavior) being the obtained value at the end of the process:

$$\nu_{xy} = -\frac{\varepsilon_x}{\varepsilon_y} \Rightarrow \nu_{xy} = -0.51$$

5. Concluding Remarks

- Using the Finite Element Method, full 3-D numerical models of different types of multistable metastructures (row-type, plate-type and cylinder-type) have been developed and validated with experimental results from the literature, achieving relative errors lower than 10 %.
- A parametric analysis of the influence of certain geometrical factors on the energy absorption capacity has been carried out, showing that the Internal Energy increases with the increment of the number of the number of heights, the number of unit cells per side, the number of sides in a cylinder-type metastructure and the unit cell thickness (being a non-linear proportionality correlation in the latter case).
- The inertial and wave propagation effects characteristic of dynamic conditions, result in a sequential collapse of the different heights (levels) of the structures. The successive collapses do not always occur between contiguous levels.
- The auxetic behavior of plate-type metastructures has been analyzed and reproduced, obtaining an apparent Poisson's Coefficient value at the end of compression process of $\nu_{xy} = -0.51$.

References

[1] S. Shan, S.H. Kang, J.R. Raney, P. Wang, L. Fang, F. Candido, J.A. Lewis, K. Bertoldi. Multistable Architected Materials for Trapping Elastic Strain. *Advanced Materials* 2015; 27:4296-4301.

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[4] A. Muller, M.C. Wapler, U. Wallrabe. A quick and accurate method to determine the Poisson's ratio and the coefficient of thermal expansion of PDMS. *Soft Matter* 2018; 15:779-784.

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